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Compact parity-conserving percolation in one dimension

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Abstract. Compact directed percolation is known to appear at the endpoint of the directed percolation critical line of the Domany–Kinzel cellular automaton in 1 + 1 dimension. Equivalently, such transition occurs at zero temperature in a magnetic field H, upon changing the sign of H, in the one-dimensional Glauber–Ising model, with well known exponents characterizing spin–cluster growth. We have investigated here numerically these exponents in the non-equilibrium generalization of the Glauber model in the vicinity of the parity-conserving phase transition point of the kinks. Critical fluctuations on the level of kinks are found to affect drastically the characteristic exponents of spreading of spins while the hyperscaling relation holds in its form appropriate for compact clusters.

1. Introduction

In the one-dimensional (1D) Domany–Kinzel automaton (DKCA) [1, 2] the state $\sigma(i,t)$ of site i at time t depends on $\sigma(i-1,t-1)+\sigma(i+1,t-1)$, $(\sigma(i,t)=0,1)$. Of the conditional probabilities $p(\sigma(i-1,t-1)\sigma(i+1,t-1)|\sigma(i,t))$ the independent ones are denoted by $p_0=p(00|1)$, $p_1=p(01|1)=p(10|1)$ and $p_2=p(11|1)$. All sites are updated simultaneously in the process. The phase diagram of the DKCA in the (p_1,p_2) plane, exhibits a line of (second-order) critical points of directed percolation universality class, which line ends at the so-called compact directed percolation point (CDP). This point is situated on the line $p_2=1$, $p_0=0$ at $p_1=\frac{1}{2}$. By crossing this point (changing the sign of $p_1-\frac{1}{2}$) the transition is a first order one between two ordered phases (empty and full or, equivalently, using the spin variable $s(i,t)=2\sigma(i,t)-1$, all spins up and all spins down). The characteristic critical exponents of the CDP transition are known exactly and a hyperscaling relation has also been derived for such transitions in d dimensions [1, 3]. For the spreading process of a single $\sigma(i,0)=1$ in the sea of zeros the exponents δ_s, η_s and z_s defined at the transition point for the power-law time dependences of the density of 1's $n_s\propto t^{\eta_s}$, the survival probability $P_s(t)\propto t^{-\delta_s}$ and the mean square distances of spreading $\langle R_s^2(t)\rangle\sim t^{z_s}$ have been obtained as 0, $\frac{1}{2}$ and 1, respectively [3]. (In the following subscript s refers to spins for all the quantities. Without subscript the corresponding quantity for kinks is meant, except for ν.) For the parallel (time-direction) and perpendicular (space-direction) coherence lengths $\nu_\parallel$ and $\nu_\perp$ resp., as well as for the dynamical critical exponent Z Domany and Kinzel have obtained the exact results: $\nu_\parallel=2$, $\nu_\perp=1$, $Z=2$, respectively (which means only two exponents as by definition $\nu_\parallel=Z\nu_\perp$). The above-mentioned hyperscaling law [3]

$$\eta_s+\delta_s=dz_s/2 \tag{1}$$

is fulfilled with the above exponents. More generally, Dickman and Tretyakov argued, that equation (1) is valid at first order transitions and it should apply to cases, whenever power-law growth produces compact 'colonies', developing from single seeds. In [3] 'compactness' is clearly defined: it is meant that the density of colonies in surviving samples remains finite for $t \to \infty$.

It is obvious that the above-sketched $(1+1)$-dimensional CDP transition is equivalent to that in the 1D (ferromagnetic) Glauber–Ising model [4] at $T = 0$, because the symmetry as well as the kinetics are the same. Changing the parameter p_1 of the DKCA around $p_1 = \frac{1}{2}$ corresponds to introducing a magnetic field H into the spin–flip probability w_i of the Glauber–Ising model (section 2) and changing its sign.

On the basis of this equivalence it is of some interest to investigate the same spreading problem in the framework of the non-equilibrium generalization [5, 6] of the kinetic Ising model (NEKIM) (section 3), where, in some range of its parameters, there is a continuous transition between a single domain and a multidomain state. The order parameter of this transition is the density of kinks. The critical fluctuations of this so-called parity-conserving (PC) transition [7, 8, 6, 9–11] have pronounced effects on the underlying spin system, as was found earlier [12] both in case of static and dynamic exponents *in situations of quenching from* $T = \infty$ (random initial states). These investigations will now be completed by studying, via numerical simulations, the spin spreading process at the PC point (section 4). It is found that the characteristic exponents differ from those of the CDP transition, as could be expected, but basic similarities still remain. Thus, the transition which takes place upon changing the sign of the magnetic field is of first order and its exponents satisfy equation (1). Accordingly it can be called 'compact', and we will call it compact parity-conserving (CPC) transition†. The static magnetic critical exponent Δ is also determined at the PC point.

2. Glauber–Ising model

The $d = 1$ Ising model with Glauber kinetics is exactly solvable. In this case the critical temperature is at $T = 0$, the transition is of first order. We recall that $p_T = e^{-\frac{4J}{kT}}$ plays the role of $\frac{T-T_c}{T_c}$ in one dimension and in the vicinity of $T = 0$ critical exponents can be defined as powers of p_T, thus for example that of the coherence length, ν, via $\xi \propto {p_T}^{-\nu}$. In the presence of a magnetic field H, (when the Ising Hamiltonian is given by $\mathcal{H} = -J\sum_i s_i s_{i+1} - H\sum_i s_i$, $s_i = \pm 1$), the magnetization is known exactly. At $T = 0$

$$m(T = 0, H) = \mathrm{sgn}(H). \tag{2}$$

Moreover, for $\xi \gg 1$ and $H/kT \ll 1$ the the exact solution reduces to

$$m \sim 2h\xi \qquad h = H/k_B T. \tag{3}$$

In scaling form one writes:

$$m \sim \xi^{-\frac{\beta_s}{\nu}} g(h\xi^{\frac{\Delta}{\nu}}) \tag{4}$$

where Δ is the static magnetic critical exponent. Comparison of equations (3) and (4) results in $\beta_s = 0$ and $\Delta = \nu$. These values are well known for the 1D Ising model. It is clear that the transition is discontinuous at $H = 0$ also when changing H from positive to negative values, see equation (2). (In the following the order of limits will always be meant as: first $H \to 0$ and then $T \to 0$.)

† In a previous paper of the present authors [13], devoted to damage spreading investigations of different non-equilibrium one-dimensional models, the issue of a CPC transition has already been raised.

The kinetics of the Ising model in a magnetic field has been formulated by Glauber [4]. In its most general form the spin–flip transition rate for spin s_i sitting at site i is:

$$w_i{}^h = w_i(1 - \tanh hs_i) \approx w_i(1 - hs_i) \tag{5}$$

$$w_i = \frac{\Gamma}{2}(1 + \tilde{\delta}s_{i-1}s_{i+1})\left(1 - \frac{\gamma}{2}s_i(s_{i-1} + s_{i+1})\right) \tag{6}$$

where $\gamma = \tanh 2J/kT$ (J denoting the coupling constant in the Ising Hamiltonian) Γ and $\tilde{\delta}$ are further parameters. This model will reach the same equilibrium state as the Ising model in a magnetic field.

For the case $\tilde{\delta} = 0$, $\Gamma = 1.0$, which is usually referred to as the Glauber–Ising model, $Z = 2$ (Z is the usual dynamic critical exponent) is also a well known result. In section 3 we will give a brief review of the non-equilibrium generalization of the kinetic Ising model which will be used later.

3. The non-equilibrium generalization model

In the non-equilibrium generalization model (NEKIM), besides the spin–flip transition rate equation (6), taken at $T = 0$, also a nearest-neighbour mixing of spins with probability p_{ex} is applied at each timestep of the simulation. The spin–exchange transition rate of nearest-neighbour spins (the Kawasaki [14] rate at $T = \infty$) is $w_{ii+1} = \frac{1}{2}p_{ex}[1 - s_i s_{i+1}]$, where p_{ex} is the probability of spin–exchange. Spin–flip and spin–exchange are then applied alternately. The model was originally proposed and investigated for values $\tilde{\delta} \geqslant 0$ at finite temperatures in [5]. It is , however, at $T = 0$ and for negative values of $\tilde{\delta}$, that in this system a second order phase transition takes place [6] for the *kinks* from an absorbing to an active state, which belongs to the PC universality class. The order parameter is the density of kinks, at the PC point it decays in time as a power law $n_{\text{kink}} \propto t^{-\alpha}$, with $\alpha = 0.285(3)$.

The absorbing phase is double degenerate, an initial state decays algebraically to the stationary state, which is one of the absorbing ones (all spins up or all spins down, provided the initial state has an even number of kinks) and the whole absorbing phase behaves like a critical point with power-law decay of correlations, like the Glauber–Ising point ($\tilde{\delta} = 0$, $p_{ex} = 0$).

Now let us look at the PC transition from the point of view of the underlying spin system. The above-mentioned first-order transition at $T = 0$ of the Ising system disappears at the PC point and is, of course, absent in the whole active phase of the kinks. The fluctuations of this PC transition exert a pronounced effect on the underlying spin system as found earlier [12] thus, e.g. the the classical dynamical exponent Z, defined, as usual through the relaxation time τ_s of the magnetization, $\tau_s \propto \xi^{\tilde{Z}}$, was found to be $Z = 1.75(1)$ instead of the Glauber–Ising value of $Z = 2$. In this case one approaches the PC point from the temperature 'direction', by decreasing it to 0 (the effect of temperature is to create kink pairs inside of ordered spin domains). On the other hand, we can also look at the transition by changing a characteristic parameter (chosen by us to be $\tilde{\delta}$) of NEKIM through the critical point $\tilde{\delta}_c$ and fixing the other two. As a function of $\epsilon = |(\tilde{\delta} - \tilde{\delta}_c)|$ the transition—on the level of spins—is again a first-order one of type order-disorder. Namely, taking initial states with an even number if kinks, the magnetization of the stationary state has a jump at $\epsilon = 0$. The same is true when changing the magnetic field h from negative to positive values at $\epsilon = 0$. Thus for the spins the value of the (static) critical exponent β_s is zero, in all the three 'directions' of departing from PC (p_T, ϵ and h), as mentioned above. (For simulational results see [12, 13].)

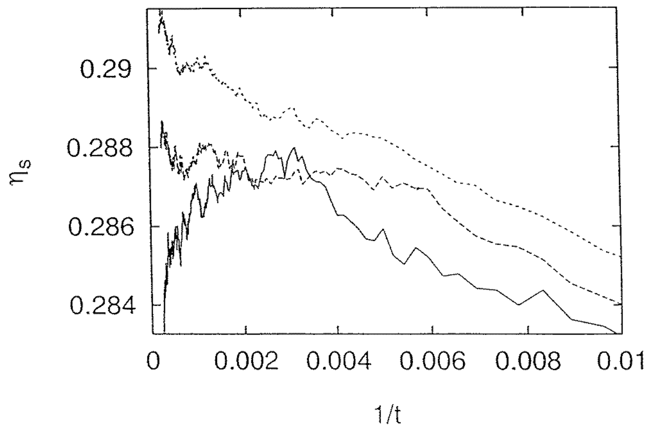

Figure 1. Local slopes of the spin density $n_s(t)$ for zero magnetic field near the PC point. $-\tilde{\delta} = 0.393, 0.394, 0.395$ (from bottom to top). The best scaling result is $\eta_s = 0.288(4)$. In the averaging the number of independent runs was 3–5 $\times 10^6$.

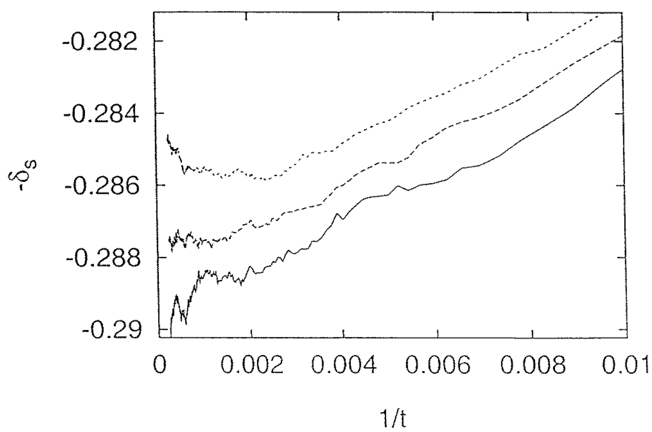

Figure 2. Local slopes of the survival probability $P_s(t)$ for zero magnetic field in the vicinity of the PC point. $-\tilde{\delta} = 0.393, 0.394, 0.395$ (from bottom to top). The best scaling result is $\delta_s = 0.287(3)$. In the averaging the number of independent runs was the same as for figure 1.

In the following we will choose the same PC transition point as in previous works [6, 12], and make simulations at and around this point by changing the magnetic field h. The parameters chosen are: $\Gamma = 0.35$, $p_{ex} = 0.3$, $\tilde{\delta}_c = -0.395(2)$. In these previous simulations the spin–flip part has been applied using two-sublattice updating. After that we have stored the states of the spins and made L (L is the size of the system) random attempts of exchange using always the stored situation for the states of the spins before updating. Together all these have been counted as one timestep of updating. (Usual Monte Carlo update in this last step enhances the effect of p_{ex} and leads to $\tilde{\delta}_c = -0.362(1)$.)

4. Spin-cluster-growth simulations

Time-dependent simulations have proven to be a very efficient method for determining critical exponents (besides the critical point itself) [15–17]. On the basis of equation (4),

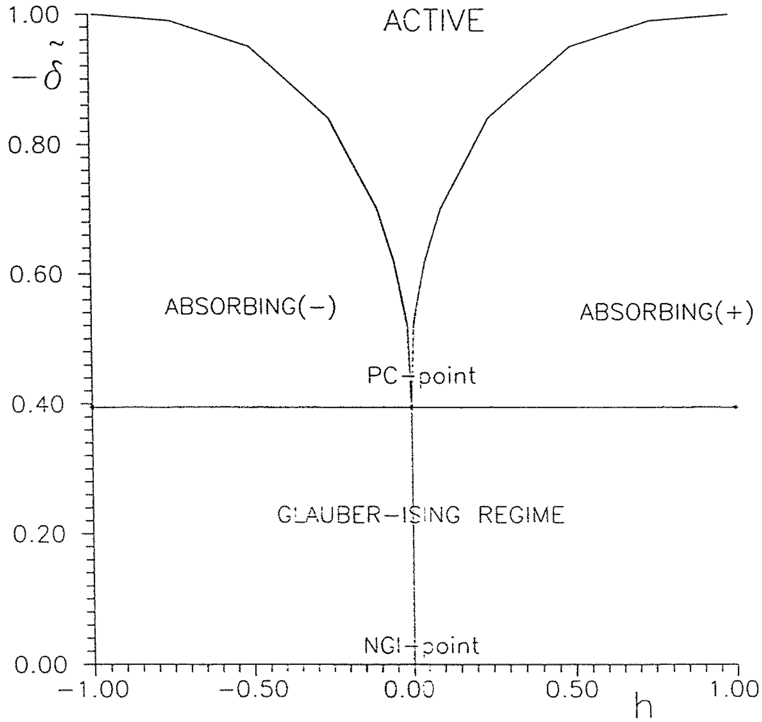

Figure 3. Phase diagram of NEKIM in the $(h, -\tilde{\delta})$ plane. The chosen PC-point is at $\tilde{\delta} = -0.395$. For lower values of $-\tilde{\delta}$, in the Glauber–Ising regime, the vertical line connecting the PC and the NGI points (at $h = 0.0$) consists of all CDP points with its characteristic critical exponents. The simulations around the PC point have been done here for $h > 0$ in the interval $0 \leqslant h \leqslant 0.1$. The other parameters of NEKIM in the whole plane are as follows: $\Gamma = 0.35$ and $p_{ex} = 0.3$.

the t-dependence of the magnetization in scaling form can be written as

$$m(t, h) \sim t^{-\frac{\beta_s}{\nu Z}} \tilde{g}(h t^{\frac{\Delta}{\nu Z}}). \tag{7}$$

Such a form can be used in a quench from $T = \infty$ to T_c and was exploited also in [12], though at $h = 0$, using temperature as a second variable, for determining mainly static critical exponents of the spins at the PC point.

In the following we will further study the influence of the PC transition on the spin system from a different point of view. Instead of starting with an initial state of randomly distributed up- and down-spins with zero average magnetization as in the above-mentioned simulations of quenching, we will now investigate the evolution of the non-equilibrium system from an almost perfectly magnetized initial state (or rather an ensemble of such states). This state is prepared in such a way that a single up-spin is placed in the sea of down-spins at $L/2$. Using the language of kinks (or particles, in the branching annihilating random walk (BARW) model [18, 9]) this corresponds to the usual initial state of two nearest-neighbour kinks placed at the origin. The quantities usually measured of the forming clusters are the order-parameter density, the survival probability $P(t)$ and the average mean square size of spreading $\langle R^2(t) \rangle$ from the centre of the lattice. At the critical point these quantities exhibit power-law behaviour in the limit of long times; more generally we can write

$$n_s(t, h) \sim t^{\eta_s} g_1(h t^{\frac{\Delta}{\nu Z}}) \tag{8}$$

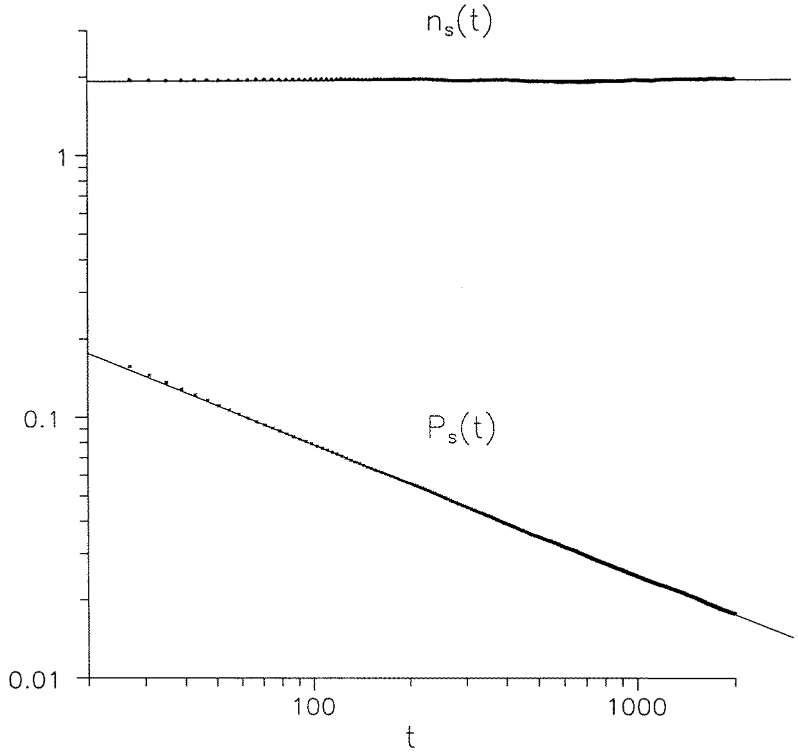

Figure 4. The scaling function $n_s(t) \propto t^{\eta_s}$ and $P_s(t) \propto t^{-\delta_s}$ at the NGI point of figure 3 ($p_{ex} = 0.3$, $\tilde{\delta} = 0$). The power-law fit of the data shown gives ${\eta_s}^{\mathrm{NGI}} = 0.0006$ and ${\delta_s}^{\mathrm{NGI}} = 0.500(5)$. Number of independent runs in the averaging was (10–60) $\times 10^4$.

for the deviation of the spin density from its initial value, $n_s = m(t, h) - m(0)$,

$$P_s(t, h) \sim t^{-\delta_s} g_2(ht^{\frac{\Delta}{\nu Z}}) \tag{9}$$

for the survival probability and

$$\langle {R_s}^2(t, h)\rangle \sim t^{z_s} g_3(ht^{\frac{\Delta}{\nu Z}}) \tag{10}$$

for the average mean square distance of spreading from the origin. The argument of the scaling functions above has been taken from equation (7); but now at the PC point instead of the Glauber–Ising one. Thus the exponents Δ, ν and Z in the above equations take values appropriate at the PC point. We note here that the coherence length exponent ν appearing above is basically different from the $\nu_\perp$ and $\nu_\|$ generally used in the context of directed percolation (DP) transitions or in connection with the *kinks* in NEKIM. Namely, $\xi_\perp \propto \epsilon^{-\nu_\perp}$ with ϵ denoting the deviation from the PC point in the 'direction' of the quantity driving the phase transition ($\epsilon = |\tilde{\delta} - \tilde{\delta}_c|$ here). Moreover, $\nu_\| = \nu_\perp Z$. (Z is, of course, independent of the above-mentioned 'directions' [12].)

We have measured $n_s(t)$ and $P_s(t)$ at and in the vicinity of the critical point with initial configuration of a single up-spin at the origin in the sea of down-spins and allowing the system to evolve according to the rule of NEKIM as described above. Averaging has been taken over runs with different sequences of random numbers during the evolution. Figures 1 and 2 show the local slopes (for a definition see, e.g. [9]) η_s and $-\delta_s$, respectively. As the survival probability must be the same for spins and kinks (if the minority spin dies

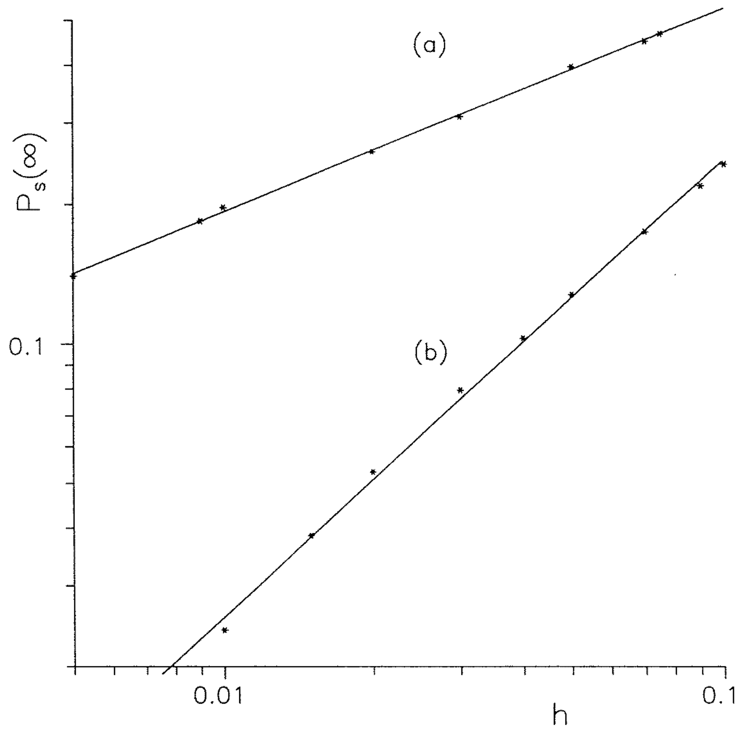

Figure 5. Level-off values of the survival probability $P_s(t, h)$ for large times at different values of h. The straight line (a) is power-law fit near the PC point with $\beta_s{}' = 0.445(5)$. Points around the straight line (b) show simulational results of the same quantity for the NGI point, giving $\beta_s{}'^{\text{NGI}} = 0.99(2)$. Number of independent runs in the averaging was 10^4–10^5.

out, kinks also disappear and vice versa) $\delta_s = \delta$. The same applies also for the root mean square size of the cluster. As no result has been reported before for δ in the NEKIM model, exhibiting figure 2 has its own merits. η_s, however, is an independent new exponent.

(Besides results at the PC point we have also carried out detailed simulational studies at a point in the so-called Ising phase, namely for $\tilde{\delta} = 0$, $\Gamma = 0.35$ and $p_{ex} = 0.3$. This point is a non-equilibrium one due to the non-zero value of p_{ex}, and is marked on figure 3 with NGI (non-equilibrium Glauber–Ising) on the abscissa. The results which we have obtained via simulations at this point (figure 4) are, within error, the same as for the (exactly solved) Glauber–Ising case.)

Figure 5 shows the the asymptotic values for large times of $P_s(t, h)$, for different values of h in the range of $h = 0.005 - 0.1$. For the exponent $\beta_s{}'$ defined through

$$\lim_{t\to\infty} P_s(t, h) \propto h^{\beta_s{}'} \tag{11}$$

the value $\beta_s{}' = .445(5)$ has been obtained. Figures 6 and 7 show the scaling functions, equations (8) and (9), respectively. Here the best fit for the scaling together of data with different values of h could be achieved with $\Delta = 0.49(1)$, using the measured values $\delta_s = 0.285$, $\eta_s = 0.285$, and that of νZ from former studies, $\nu Z = 0.777$ [12]. Data for different values of h scale together sufficiently well when considering the relatively poor statistics (averages over 4×10^4 samples, typically).

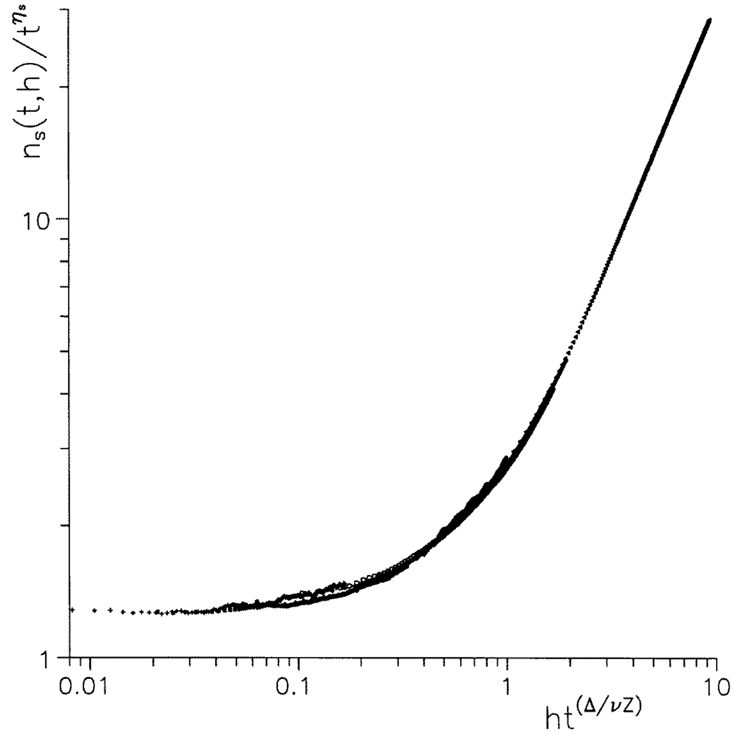

Figure 6. The scaling function $n_s(t, h)$. The different curves correspond to the following values of the parameter h: $h = 0.05, 0.01, 0.009, 0.003, 0.001, 0.0005$. The values of the parameters η_s, Δ and νZ are given in the text. Number of independent runs in the averaging was 4×10^4.

The scaling law

$$\beta_s{}' = \frac{\delta_s \nu Z}{\Delta} \tag{12}$$

following from equation (9) is satisfied with the above values of the exponents, within error. We note here that $\beta_s{}'$ can be connected with $\beta_{\mathrm{kink}} = \beta_{\mathrm{kink}}{}'$, using in equation (12) $\delta_s = \delta$ and the definition of $\beta_{\mathrm{kink}}{}'$ from [19] with the result: $\beta_s{}' = \beta_{\mathrm{kink}} \nu/(\nu_\perp \Delta)$.

The hyperscaling law for the spreading exponents was derived in its most general form by Mendes *et al* [19] which we write here for the spin quantities:

$$\left(1 + \frac{\beta_s}{\beta_s{}'}\right) \delta_s + \eta_s = dz_s/2. \tag{13}$$

In equation (13), in analogy with the spin-cluster-growth description at and in the vicinity of the CDP transition of the DKCA [3], the above finite value of $\beta_s{}'$ enters. (As explained in the introduction, $(p_1 - p_{1c})$ of the DKCA, with $p_{1c} = \frac{1}{2}$, corresponds to h in the Glauber–Ising formulation.) Moreover, $\beta_s = 0$, which value follows near the PC point from the same symmetry consideration as at the Glauber–Ising point (though does not in the active phase). Here, again, one should recall the above-mentioned analogy between the DKCA's variable $(p_1 - \frac{1}{2})$ and the variable h in this case. With the exponents obtained and summarized in table 1 equation (13) is fulfilled. As already mentioned in the introduction, according to the argumentation of [3] the fulfilment of the hyperscaling law in the above form is equivalent to compactness of the clusters. For illustration developing clusters are exhibited on figure 8

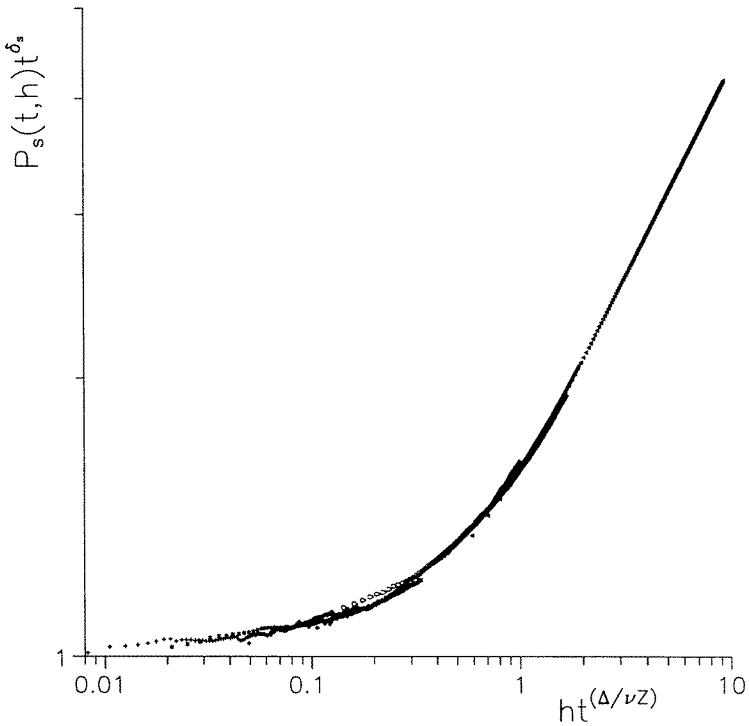

Figure 7. The scaling function $P_s(t,h)$. The values of the parameters η_s, Δ and νZ used are given in the text. Number of independent runs and values of h are the same as for figure 6.

Table 1. Spin-cluster critical exponents for NEKIM in a magnetic field.

	β_s	ν	$\beta_s{}'$	Δ	η_s	δ_s	z_s
NGI–CDP	0	$\frac{1}{2}$	0.99(2)	$\frac{1}{2}$	0.0006(4)	0.500(5)	$1(=2/Z)$
CPC	0.00(2)	0.444	0.45(1)	0.49(1)	0.288(4)	0.287(3)	$1.14(=2/Z)$

under three conditions: (a) Glauber case (CDP in the DKCA sense), (b) at the NGI point (see figure 3) where the kinetics is a non-equilibrium one ($p_{ex} \neq 0$) and (c) at the PC point. It is apparent that the minority phase never develops inside of the majority one, moreover, the branching process present in the kinetics in cases (b) and (c) makes the flat pieces of CDP boundaries fringed.

The results together with some of the critical exponents obtained earlier in [12] are summarized in table 1.

5. Summary

In summary, we have carried out numerical studies of the power-law behaviour of spreading of spins, at the PC transition point of NEKIM (where a second-order transition occurs on the level of *kinks*). It has been found that the analogue of the Domany–Kinzel CDP transition—a first-order transition upon changing the sign of an applied magnetic field—still exists. Of the three exponents measured only Δ, which is the static magnetic exponent of the Ising

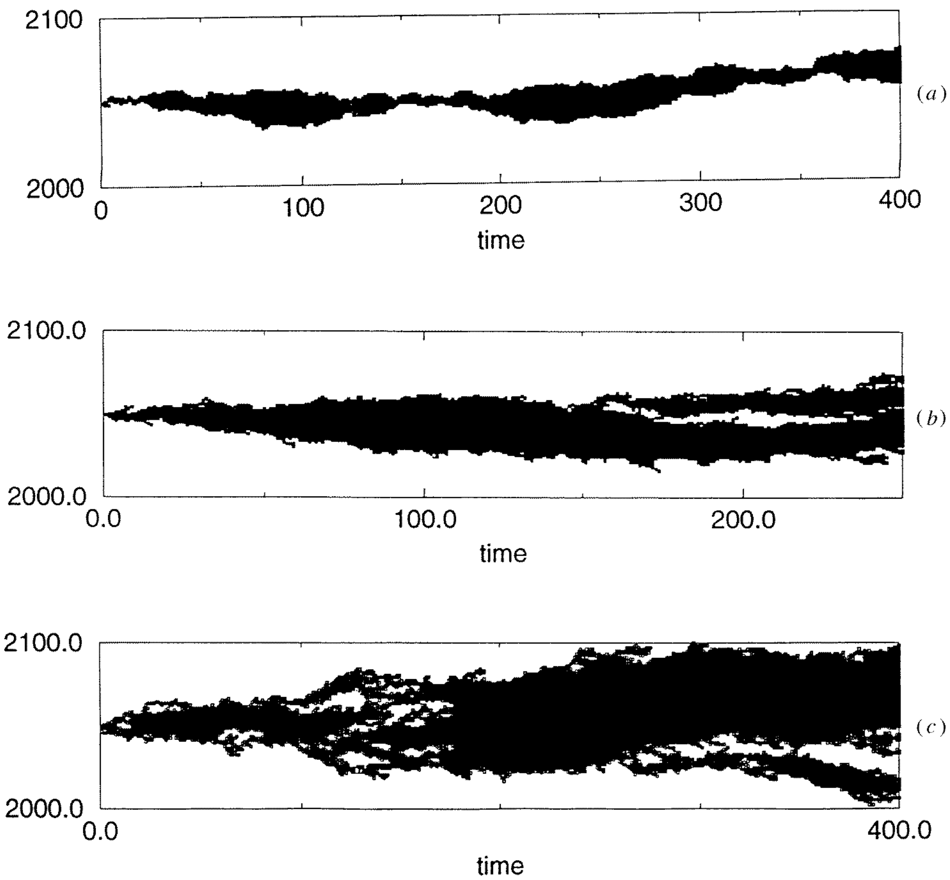

Figure 8. Clusters developing from a single up-spin (dark point) in the sea of down-spins (white points) at $t = 0$ for three choices of NEKIM parameters, see text.

model, was found to be unaffected by the critical fluctuations of the kinks within error. In view of other static Ising exponents, this circumstance is not so natural, such as the coherence length exponent is a counterexample (see [12] and table 1). Consequently the relation $\Delta = \nu$ valid in the Glauber–Ising case is no more fulfilled at the PC point. $\beta_s{}'$ characterizing the level-off values of the survival probability of the spin clusters is a new (static) exponent; Δ and $\beta_s{}'$ are connected by a scaling law. The third exponent, η_s has proven to be numerically equal to $\delta_s = \delta$ thus ensuring that the hyperscaling law is fulfilled in a form appropriate for first order transitions and compact clusters [19, 3]. Moreover, we have reported results of simulation for exponent δ in case of NEKIM for the first time.

These results give further evidence to the conclusion that the effect of fluctuations felt by the spin system at the PC transition is of interest in itself.

Acknowledgments

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